STRUCTURE OF NEUTRON STAR CORES*

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Abstract. After reviewing the outer and central regions of a neutron star, we discuss the central region and the possibility that the core has a solid structure. We present the work of different groups on the solidification problem, suggesting that the neutron star-cores are indeed solid.

1. Introduction

Neutron stars are believed to be formed in the gravitational implosion of aged, massive stars which have used up their thermonuclear energy. Matter in such stars is so dense that the gravitational attraction is balanced by the pressure of the highly-degenerate neutrons. This requires densities of the order $\sim 10^{14}$ g cm^{-3}. Since stellar masses are comparable to the solar mass ($= 2 \times 10^{33}$ g), the radius of a neutron star may be expected to be of the order ~ 10 km. The cross-section of a typical medium-weight neutron star is shown in Figure 1.

It has been suggested that matter at high densities, such as found in neutron star cores, has a crystalline structure. After briefly describing the crust and central region of a neutron star, we discuss the core region and the possibility for a crystalline structure to exist.

2. The Crusts and the Central Region

The main constituents of the outer crust are Fe^{56} nuclei (the end point of thermonuclear burning) and a gas of free degenerate electrons, produced because of pressure ionization. These nuclei, embedded in a gas of freely-moving and relativistic electrons, are only weakly screened by the electrons, and as Ruderman (1968) first suggested, they arrange themselves in a crystalline structure.

As the density increases, the electron chemical potential also increases so that the process of inverse beta-decay becomes energetically favorable, i.e.,

$$(Z, A) + e^- \rightarrow (Z - 1, A) + \nu .$$

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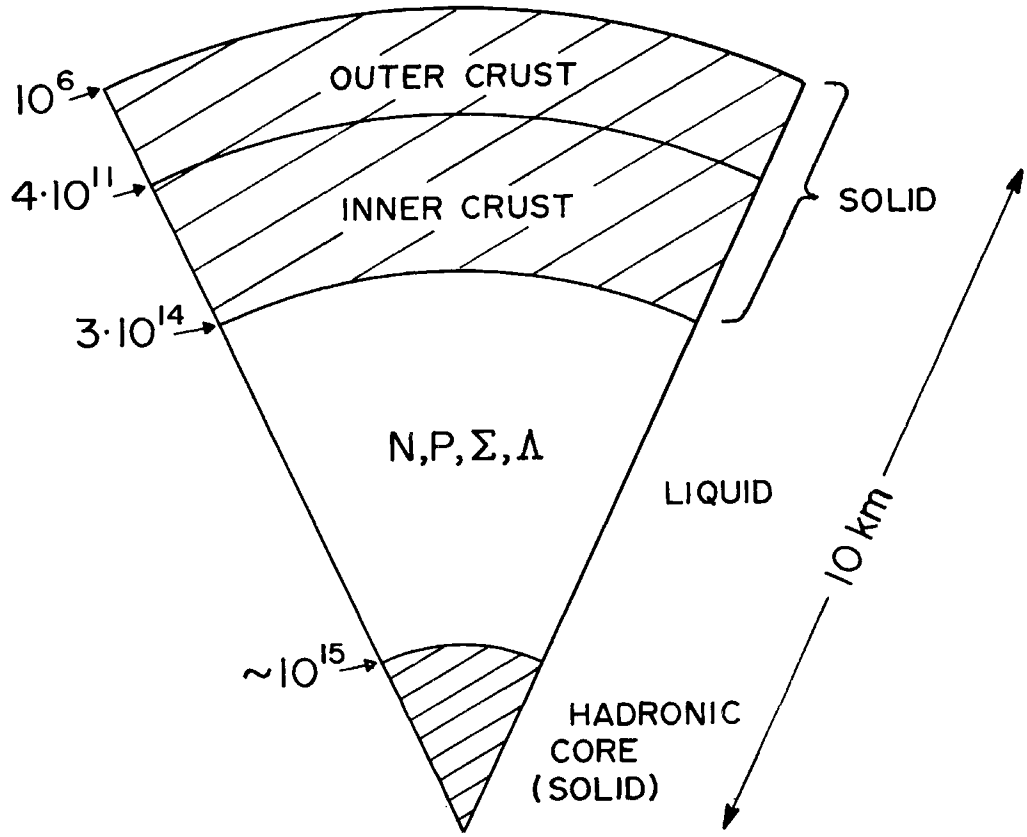

Fig. 1. Schematic representation of a neutron star interior. The numbers on the left are the densities (in g cm^{-3}) at which the several phase-transitions occur.

This process produces nuclei which are more and more neutron-rich, and at the same time stable against beta-decay, because of the presence of the degenerate electron gas. At a density $\sim 4 \times 10^{11}$ g cm^{-3}, the neutron excess reaches a point where the last neutron is no longer bound. The neutrons then start to drip out of the nuclei and the system is now composed of nuclei, electrons and free neutrons. This transition, known as the Neutron Drip Point, marks the separation of the inner crust from the outer crust. As the density increases, the protons within the nuclei find it energetically more favorable to spread around uniformly rather than be clumped together in the nuclei. The nuclei become less and less localized in space, and finally dissolve in the surrounding medium. The region beneath the inner crust, the central region, is thus composed of a mixture of three degenerate quantum liquids: neutrons, protons and electrons. It is expected that the neutrons and the protons can be in superfluid states, while the electrons remain a normal, weakly interacting relativistic plasma.

3. The Core Region and the Solidification Problem

As the density exceeds the nuclear matter density, one reaches the so-called hadronic core wherein mesons and hyperons are expected to be present in addition to neutrons, protons and electrons. The calculation of the equation of state for this quantum system of baryons is a formidable task because it needs a knowledge of the hyperon-nucleon and hyperon-hyperon interactions as well as that of a reliable many-body technique.

An important question is whether the neutron matter inside the neutron star core will turn into a quantum crystal. The problem has been investigated by several groups over the last few years, and most of them suggest that solidification of neutron matter will take place at such high densities. There is, however, no consensus as to the exact value of the solidification density which varies from 4.2×10^{14} g cm^{-3} to $(2.9\pm0.5)\times 10^{15}$ g cm^{-3}. The problem is complicated by the fact that one does not know, as yet, the behaviour of N-N interaction near the origin. We discuss below the work that has so far been done on this solidification problem.

The old hard-core N-N potential almost by definition leads to a solid structure at particle densities of the order of $n_B=(\frac{4}{3}\pi r_c^3)^{-1}$, because at these densities every particle feels an infinite repulsion due to every other particle around it. This leads to a localization of the particles. With soft-core N-N potentials, however, one cannot make such definitive conclusions without first performing a microscopic calculation.

The first workers who pointed out that physically the solid state could be a more preferable configuration for neutron matter than a liquid state were Cazzola *et al.* (1966), who obtained the equation of state by solving the Dirac equation for each nucleon, assumed to be moving in a square-well potential. Their work, however, was inconclusive in the sense that they did not actually show that a crystal structure indeed occurred. Anderson and Palmer (1971) suggested that one could draw a possible analogy between a neutron liquid and He^3 liquid (see also Palmer and Anderson, 1974), and calculate the solidification density by applying the so-called law of corresponding states. Their calculations predicted the solidification density to be 5×10^{14} g cm^{-3}. Although the analogy between neutron liquid and He^3 can be instructive, it must be pointed out that it is not entirely correct, because the stiffness of the Lennard-Jones 6–12 potential, due to the term r^{-12}, has no analog in the presently known nucleon-nucleon potentials, whose forms tend to be somewhat softer. Moreover, the Lennard-Jones potential is spherically symmetric whereas the N-N potential has important non-central components like spin and angular momentum dependences, which cannot be ignored in realistic calculations. An attempt employing the Hartree-Fock variational method was tried by Coldwell (1972) who used the Reid soft-core potential for the N-N interaction and found that nucleons become localized at densities higher than $\sim7\times10^{14}$ g cm^{-3}. However his work is incomplete in the sense that the trial wave function he used does not take into account the distortion of the two-body wave-function when the relative separation of the two particles becomes small. Pandharipande (1973) tackled the solidification problem by expanding the ground state energy in clusters up to second order and then minimizing the energy. The correlated 2-body wave-function was, in turn, found by solving the homogeneous Bethe-Goldstone equation. The basic idea of his computation, which is called lowest order constrained variation (LOCV), is to put restrictions on the correlation function (which is the ratio of the correlated 2-body wave function to the uncorrelated 2-body wave function) so as to make the truncated cluster expansion a reasonable one. Pandharipande's treatment of the Bethe-Goldstone equation is, however, not fully convincing. Firstly, we should note that the interaction Hamiltonian employed in the study of quantum crystals is not invariant under the transformation $\mathbf{r} \to -\mathbf{r}$. This implies that the wave-function does not possess spatial symmetry as regards the angular-momentum decomposition. Consequently, all the angular momentum components of the wave-function are coupled. This makes the problem rather complicated because at these high densities one has to include interactions up to 5 or 6 partial waves. This, in turn, gives rise to about 20 to 24 coupled differential equations. The coupling terms are not trivial and the simple decoupling of all the waves, as performed by Pandharipande in his work, leaves out many interesting features of the problem. His results indicate that the neutron matter will not solidify until a very high density ($\sim 8 \times 10^{15}$ g cm^{-3}).

Canuto and Chitre (1973, 1974) developed an extensive and thorough way of handling the Bethe-Goldstone equation, without ignoring the coupling of all partial waves and using the most general form of the nucleon-nucleon potential. Once the wave functions are obtained by solving the Bethe-Goldstone equation, they can be used in either the variational approach or the t-matrix method. Canuto and Chitre used the t-matrix approach. The variational method has the shortcoming that it cannot handle the spin and angular momentum dependences of nuclear forces in a straight-forward manner. It has the advantage that it enables one to take into account the higher-order many-body contributions by putting certain restrictions on the correlation function. However, if the effects of higher-order correlations are not important, then the t-matrix method is better because it can fully handle, to any degree of accuracy, the state dependence of the nuclear forces. Considering a system of nucleons, the hamiltonian for the system is

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \frac{1}{2}\sum_{i<j} V_{ij}. \tag{1}$$

The Slater determinant for the system was built up of single-particle wave functions of the gaussian form

$$\phi(i) = \frac{\alpha^{3/2}}{\pi^{3/4}} e^{-\alpha^2/2|\mathbf{r}_i - \mathbf{R}_i|^2}, \quad \alpha^2 = \frac{m\omega}{\hbar}, \tag{2}$$

where R_i is the ith lattice site around which the particle performs an oscillatory motion due to the influence of the remaining $(N-1)$ particles. The t-matrix expansion gives the following expression for the energy per particle (up to and including 2-body clusters)

$$\begin{aligned}\frac{E}{N} &= \tfrac{3}{4}\hbar\omega + \sum_{i,j} \frac{\int \psi_{ij}^* V_{ij}\phi_{ij}\, \mathrm{d}^3r_i\, \mathrm{d}^3r_j}{2N\int \psi_{ij}^*\phi_{ij}\, \mathrm{d}^3r_i\, \mathrm{d}^3r_j} \\ &= \tfrac{3}{4}\hbar\omega + \tfrac{1}{2}\sum_k n_k\varepsilon_k, \end{aligned} \tag{3}$$

where ϕ_{ij} = uncorrelated 2-body wave function
$= \phi(i)\phi(j)$
ψ_{ij} = correlated 2-body wave function

ψ_{ij} is found by solving the Bethe-Goldstone equation

$$\left\{-\frac{\hbar^2}{m}\nabla_r^2 + \tfrac{1}{4}m\omega^2(\mathbf{r}-\Delta)^2 + V(r)\right\}\psi(\mathbf{r}) = \{-\tfrac{3}{2}\hbar\omega - 2U(0)\}\psi(\mathbf{r}) \tag{4}$$

where $\Delta = \mathbf{R}_1 - \mathbf{R}_2$.

Equation (4) is difficult to solve because it contains the term $\mathbf{r}\cdot\Delta$ which couples even waves with odd waves. If an angular momentum expansion is made of $\psi(\mathbf{r})$, then an infinite set of coupled differential equations results. All previous work that dealt with such an equation invariably used some average over $\mathbf{r}\cdot\Delta$.

To judge the t-matrix method and the handling of Equation (4), Canuto *et al.* (1974) tested it for the case of solid He^3. The results are shown in Figure 2. For the

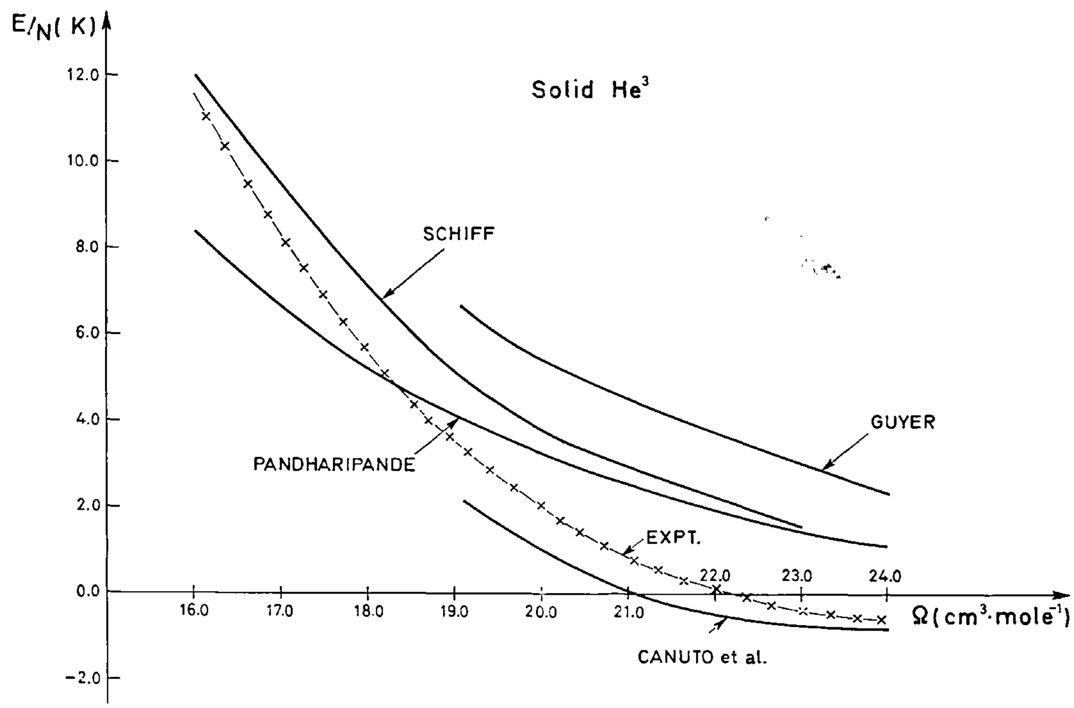

Fig. 2. Ground-state energy versus molar volume for solid He^3 as obtained by Canuto *et al.* (1974). The results of Guyer (1969), Schiff (1973) and Pandharipande (1973) are reported for comparison.

time being, significant comparison should be made between the results of Guyer (1969) who used the same method with the approximation $\mathbf{r}\cdot\Delta \simeq r\Delta$ and the results of Canuto *et al.* who expanded the wave function into its partial waves, and solved a resulting set of 25 coupled differential equations. The results of Canuto *et al.* show a significant improvement over the results of Guyer. This shows that the treatment of the Bethe-Goldstone equation as done by Canuto and Chitre is more reliable.

Now, for a complete description of the neutron matter the spin variables should be taken into account. Equation (4) then reduces to three sets of equations corresponding to $S=0$, $M_s=0$ and $S=1$ and $M_s=\pm 1$. Canuto and Chitre solved the three sets of 7, 13 and 18 coupled differential equations by considering six partial waves which they found to be large enough for the system to be stable. Their results indicate that the neutron matter will solidify at a density of about 1.6×10^{15} g cm^{-3} and that an FCC configuration is energetically more favorable than a BCC one.

Schiff (1973) has studied the solidification problem using the idea that for a high-density system of nucleons, strong repulsion is more important than the Fermi statistics, so that one can consider the Pauli principle to be a perturbation to a Bose system, for which it is easier to calculate the ground state energy. The result of his computations yields a solidification density of $(2.9\pm 0.5)\times 10^{15}$ g cm^{-3}. More recently, Nosanow and Parish (1974) used the variational approach together with Monte-Carlo techniques (to take into account the many-body effects of the short-range correlations), and obtained a solidification density at about 4.2×10^{14} g cm^{-3}, which corresponds to the region just below the superfluid region of the neutron star. However, in their investigation the spin and angular momentum dependences were treated only approximately, and they do not indicate how sensitive their results are to the choice of potentials.

The results of the calculations on the solidification problem as performed by the different groups are shown in Figure 3.

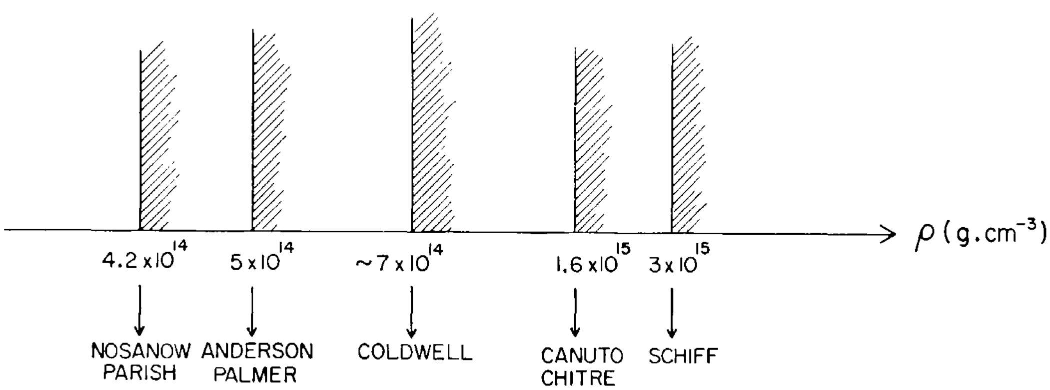

Fig. 3. Present status of the results on the solidification problem.

4. Discussion

From Figure 3, it is clear that the spread in the value of the solidification density is rather large. Hence there is considerable room for controversy about the reliability of the different calculations. From our discussions on the solidification problem, it is clear that a microscopic calculation should be the most desirable one. This makes the results of Anderson and Palmer and of Coldwell look somewhat unconvincing. The result of Noshanow and Parish, based as it is on the Monte Carlo methods, is difficult to judge in terms of a microscopic description. Besides, they do not take into account the spin and angular momentum dependences of nuclear forces explicitly. As for the computations of Schiff, one has the feeling that it does not describe a real system of neutrons in that the actual short-range repulsion is not hard-core type, as assumed by Schiff, but rather of a soft-core type. This leaves us with the only two microscopic computations by Canuto and Chitre on one hand and Pandharipande on the other hand. Canuto and Chitre's reason for adopting the t-matrix approach is based on the fact that such an approach gives good results for the test case of solid He^3 and also that it enables one to deal with the state-dependence of N-N interaction in an exact manner. On the other hand, Pandharipande's computation, though microscopic in its scope, is incomplete in that it does not treat the solution of the Bethe-Goldstone equation in a convincing manner. Perhaps the only unsatisfactory feature of Canuto and Chitre's calculations is existence of the parity-violating term $\mathbf{r}\cdot\mathbf{\Delta}$, whose implications are yet to be fully understood.

Finally, although the theoretical computation suggests that there is a very good chance that the neutron star cores will be in a solid state, the real proof has to come from astrophysical considerations. There is one piece of observational evidence which makes plausible the existence of solid cores in neutron stars. This is in reference to speed-up of the Vela pulsar. The star-quake theory (Ruderman, 1969), which quite successfully explains the speed-up of the Crab pulsar, can also explain the observational features of the Vela pulsar if the latter is assumed to have a solid core (Pines *et al.*, 1972).

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